

# Beam-plasma dielectric tensor with *Mathematica*

A. Bret

*ETSI Industriales, Universidad Castilla-La Mancha, 13071 Ciudad Real, Spain*

---

## Abstract

We present a *Mathematica* notebook allowing for the symbolic calculation of the  $3 \times 3$  dielectric tensor of a electron-beam plasma system in the fluid approximation. Calculation is detailed for a cold relativistic electron beam entering a cold magnetized plasma, and for arbitrarily oriented wave vectors. We show how one can elaborate on this example to account for temperatures, arbitrarily oriented magnetic field or a different kind of plasma.

## Program summary

*Title of program:* Tensor Mathematica.nb

*Catalogue identifier:*

*Program summary URL:*

*Program obtainable from:* CPC Program Library, Queen University of Belfast, N. Ireland

*Computer for which the program is designed and others on which it has been tested:*

*Computers:* Any computer running *Mathematica* 4.1. Tested on DELL Dimension 5100 and IBM ThinkPad T42.

*Installations:* ETSI Industriales, Universidad Castilla la Mancha, Ciudad Real, Spain

*Operating systems under which the program has been tested:* Windows XP Pro

*Programming language used:* Mathematica 4.1

*Memory required to execute with typical data:* 7.17 Mbytes

*No. of lines in distributed program, including test data, etc.:* 19

*No. of bytes in distributed program, including test data, etc.:* 4 172

*Distribution format:* .nb

*Nature of physical problem:* The dielectric tensor of a relativistic beam plasma system may be quite involved to calculate symbolically when considering a magnetized plasma, kinetic pressure, collisions between species, and so on. The present *Mathematica* Notebook performs the symbolic computation in terms of some usual dimensionless variables.

*Method of solution:* The linearized relativistic fluid equations are directly entered and solved by *Mathematica* to express the first order expression of the current. This expression is then introduced into a combination of Faraday and Ampère Maxwell's equations to give the dielectric tensor. Some additional manipulations are needed to express the result in terms of the dimensionless variables.

*Restrictions on the complexity of the problem:* Temperature effects are limited to small, i.e non-relativistic, temperatures. The kinetic counterpart of the present *Mathematica* Notebook cannot be implemented because *Mathematica* will usually not compute the required integrals.

*Typical running time:* About 1 minute on a Intel Centrino 1.5 Ghz Laptop with 512 Mo of RAM.

*Unusual features of the program:* none.

*Key words:* Plasma physics, Dielectric tensor, Fluid equations, Instabilities

*PACS:* 52.35.Qz, 52.35.Hr, 52.50.Gj, 52.57.Kk

---

*Email address:* antoineclaude.bret@uclm.es (A. Bret).

## 1 Introduction

The calculation of the dielectric tensor of a beam plasma system is a recurrent problem in plasma physics. Many efforts have been dedicated recently to such issue because of the Fast Ignition Scenario for inertial thermonuclear fusion [1,2]. According to this view, the Deuterium Tritium target is first compressed by means of some driver. Then, the compressed fuel is ignited by a relativistic electron beam generated by a petawatt laser shot. Such scenario implies therefore the interaction of a relativistic electron beam with a plasma. This kind of interaction, and its magnetized counterpart, is also relevant to astrophysics, in particular when investigating the relativistic jets of microquasars [3], active galactic nuclei [4], gamma ray burst production scenarios [5] or pulsar winds [6]. Theoretical works on these subjects are usually focused on the instabilities of the system. Although many of them demands a kinetic treatment to be fully described, the fluid equations can set some very relevant guidelines, especially when the system is not too hot. Furthermore, it has been known for long that in the relativistic regime, instabilities with arbitrarily orientated wave vectors may be essential [7,8,9,10,11]. One can therefore figure out how some refined kinetic theory may lead to almost unsolvable calculations whereas the fluid formalism is still tractable. For example, a detailed description of the collisional filamentation instability ( $\mathbf{k} \perp \text{beam}$ ) including the movement of the background ions plasma, and accounting for temperatures, was first performed through the fluid equations [12]. The very same equations were used to explore the growth rate of unstable modes with arbitrarily oriented wave vectors (with respect to the beam) when a relativistic electron beam enters a plasma [7,9,10,11]. The results were found crucial as it was demonstrated

that the fastest growing modes were indeed found for obliquely propagating waves, and the kinetic counterpart of these models has only been considered very recently [13,14,15]. As far as the magnetized case is concerned, the kinetic formalism has been thoroughly investigated for wave vectors parallel and normal to the beam [16,17,18]. But the unstable oblique modes, which once again turn to be the most unstable in many cases, could only be explored through the fluid formalism [8].

It has been demonstrated that the fluid equations yield the same first order temperature corrections than the kinetic theory for oblique modes, and the roles of both beam and plasma parallel and perpendicular temperatures are retrieved [19]. The fluid approximation is thus definitely a tool of paramount importance to deal with beam plasma instabilities. Additionally, it generally yields a polynomial dispersion equation for which numerical resolution is immediate. Nevertheless, even the fluid tensor can be analytically involved when considering arbitrarily oriented wave vectors, a guiding magnetic field, temperatures, and so on [20]. Indeed, one can think about any model based on whether the system is relativistic or not, collisional or not, magnetized or not, hot or cold. . . Most of these models have not been implemented yet, and each one should leave a quite complicated dielectric tensor.

This is why a *Mathematica* notebook has been developed which allows for the symbolic calculation of the fluid tensor, once the parameters of the system have been set. The basic system we study here is a cold relativistic electron beam entering a cold magnetized plasma with return current. As the reader shall check, the notebook is very easy to adapt the different scenarios (ion beam, temperatures, pair plasma...). The paper is structured as follow: we start introducing the theory leading to the fluid dielectric tensor in section 2.

The *Mathematica* notebook is then explained step by step in section 3, and we show how it can be modified to include temperatures or collisions before the comments and conclusion section.

## 2 Theory

We consider a beam of density  $n_b$ , velocity  $\mathbf{V}_b$  and relativistic factor  $\gamma_b = 1/(1 - V_b^2/c^2)$  entering a plasma of density  $n_p$ . Ions from the plasma are considered as a fixed neutralizing background, and an electron plasma return current flows at velocity  $\mathbf{V}_p$  such as  $n_p \mathbf{V}_p = n_b \mathbf{V}_b$ . The system is thus charge and current neutralized. We do not make any assumptions on the ratio  $n_b/n_p$  so that the return current can turn relativistic for beam densities approaching, or even equalling, the plasma one. We set the  $z$  axis along the beam velocity and align the static magnetic field along this very axis. The wave vector investigated lies in the  $(x, z)$  plan without loss of generality [8], and we define the angle  $\theta$  between  $\mathbf{k}$  and  $\mathbf{V}_b \parallel \mathbf{B}_0 \parallel z$  through  $k_z = k \cos \theta$  and  $k_x = k \sin \theta$ . The dielectric tensor of the system is obtained starting with the fluid equations for each species  $j = p$  for plasma electrons and  $j = b$  for the beam ones,

$$\frac{\partial n_j}{\partial t} - \nabla \cdot (n_j \mathbf{v}_j) = 0, \quad (1)$$

$$\frac{\partial \mathbf{p}_j}{\partial t} + (\mathbf{v}_j \cdot \nabla) \mathbf{p}_j = q \left( \mathbf{E} + \frac{\mathbf{v}_j \times \mathbf{B}}{c} \right), \quad (2)$$

where  $\mathbf{p}_j = \gamma_j m \mathbf{v}_j$ ,  $m$  the electron mass and  $q < 0$  its charge. The equations are then linearized according to a standard procedure [8], assuming small variations of the variables according to  $\exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t)$ . With the subscripts 0 and 1 denoting the equilibrium and perturbed quantities respectively, the

linearized conservation equation (1) yields

$$n_{j1} = n_{j0} \frac{\mathbf{k} \cdot \mathbf{v}_{j1}}{\omega - \mathbf{k} \cdot \mathbf{v}_{j0}}, \quad (3)$$

and the force equation (2) gives,

$$\begin{aligned} & im\gamma_j(\mathbf{k} \cdot \mathbf{v}_{j0} - \omega) \left( \mathbf{v}_{j1} + \frac{\gamma_j^2}{c^2} (\mathbf{v}_{j0} \cdot \mathbf{v}_{j1}) \mathbf{v}_{j0} \right) \\ &= q \left( \mathbf{E}_1 + \frac{(\mathbf{v}_{j0} + \mathbf{v}_{j1}) \times \mathbf{B}_0 + \mathbf{v}_{j0} \times \mathbf{B}_1}{c} \right), \end{aligned} \quad (4)$$

where  $i^2 = -1$ . Through Maxwell-Faraday equations, the field  $\mathbf{B}_1$  is then replaced by  $(c/\omega)\mathbf{k} \times \mathbf{E}_1$  so that the perturbed velocities  $\mathbf{v}_{j1}$  can be explained in terms of  $\mathbf{E}_1$  resolving the tensorial equations (4). Once the velocities are obtained, the perturbed densities can be expressed in terms of the electric field using Eqs. (3). Finally, the linear expression of the current is found in terms of  $\mathbf{E}_1$  through,

$$\mathbf{J} = q \sum_{j=p,b} n_{j0} \mathbf{v}_{j1} + n_{j1} \mathbf{v}_{j0}, \quad (5)$$

and the system is closed combining Maxwell Faraday and Maxwell Ampère equations,

$$\frac{c^2}{\omega^2} \mathbf{k} \times (\mathbf{k} \times \mathbf{E}_1) + \mathbf{E}_1 + \frac{4i\pi}{\omega} \mathbf{J} = 0. \quad (6)$$

Inserting the current expression from Eq. (5) into Eq. (6) yields an equation of the kind  $\mathcal{T}(\mathbf{E}_1) = 0$ , and the dispersion equation reads  $\det \mathcal{T} = 0$ .

The Mathematica notebook we describe in the next section performs a symbolic computation of the tensor  $\mathcal{T}$  and the dispersion equation  $\det \mathcal{T} = 0$ , in

terms of the usual [21] reduced variables of the problem

$$\mathbf{Z} = \frac{\mathbf{k}V_b}{\omega_p}, \quad x = \frac{\omega}{\omega_p}, \quad \alpha = \frac{n_b}{n_p}, \quad \beta = \frac{V_b}{c}, \quad \Omega_B = \frac{\omega_b}{\omega_p}, \quad (7)$$

where  $\omega_p^2 = 4\pi n_p q^2/m$  is the electron plasma frequency and  $\omega_b = |q|B_0/mc$  the electron cyclotron frequency.

### 3 *Mathematica* implementation

For the most part, *Mathematica* is used to solve the tensorial equations (4) for  $\mathbf{v}_{j1}$  and extract the tensors  $\mathcal{T}$  from Eqs. (5,6). We start declaring the variables corresponding to the wave vector, the electric field, the beam and plasma drift velocities and the magnetic field,

```
In[1]:= k = {kx, 0, kz}; E1 = {E1x, E1y, E1z}; V0b = {0, 0, Vb};
V0p = {0, 0, Vp}; B0={0, 0, m c ωb/q}; B1 = c Cross[k, E1]/ω;
vb1 = {vb1x, vb1y, vb1z}; vp1 = {vp1x, vp1y, vp1z};
```

Note that Maxwell Faraday's equation is already implemented in the definition of **B1**. The wave vector has no component along the  $y$  axis and the beam and plasma drift velocities only have one along the  $z$  axis. The guiding magnetic field is set along  $z$  and defined in terms of the cyclotron frequency  $\omega_b$ . This will be useful later when introducing the dimensionless parameters (7).

We then have *Mathematica* solve Eqs. (4) for the beam and the plasma. The left hand side of the equation is not as simple as in the non-relativistic case because the  $\gamma$  factors of the beam and the plasma modify the linearization procedure. We write this part of the equations in a tensorial form in *Mathe-*

*matica* defining the tensors **Mp** and **Mb** such as “left hand side”=**Mj**<sup>-1</sup>.**vj1** with,

$$In[2]:= \text{Mb} = \{\{\frac{i}{\gamma b(\omega - kzVb)}, 0, 0\}, \{0, \frac{i}{\gamma b(\omega - kzVb)}, 0\}, \{0, 0, \frac{i}{\gamma b^3(\omega - kzVb)}\}\};$$

$$In[3]:= \text{Mp} = \{\{\frac{i}{\gamma p(\omega - kzVp)}, 0, 0\}, \{0, \frac{i}{\gamma p(\omega - kzVp)}, 0\}, \{0, 0, \frac{i}{\gamma p^3(\omega - kzVp)}\}\};$$

where  $i^2 = -1$ . The reader will notice that relativistic effects are more pronounced in the beam direction due to the  $\gamma^3$  factors in the  $zz$  component. We now have *Mathematica* solve the tensorial Eqs. (4). For better clarity, we first define them

$$In[4]:= \text{EqVb} = \text{vb1} \cdot \text{Dot}[\text{Mb}, \frac{\mathbf{a}}{\mathbf{m}} \left( \mathbf{E1} + \frac{\text{Cross}[\mathbf{V0b} + \text{vb1}, \mathbf{B0}]}{c} \right)] - \text{Dot}[\text{Mb}, \frac{\mathbf{a}}{\mathbf{m}} \left( \frac{\text{Cross}[\mathbf{V0b}, \mathbf{B1}]}{c} \right)];$$

$$In[5]:= \text{EqVp} = \text{vp1} \cdot \text{Dot}[\text{Mp}, \frac{\mathbf{a}}{\mathbf{m}} \left( \mathbf{E1} + \frac{\text{Cross}[\mathbf{V0p} + \text{vp1}, \mathbf{B0}]}{c} \right)] - \text{Dot}[\text{Mp}, \frac{\mathbf{a}}{\mathbf{m}} \left( \frac{\text{Cross}[\mathbf{V0p}, \mathbf{B1}]}{c} \right)];$$

before we solve them,

$$In[6]:= \text{Vb1} = \text{FullSimplify}[\text{vb1} /. \text{Solve}[\text{EqVb} == 0, \text{vb1}][[1]]];$$

$$In[7]:= \text{Vp1} = \text{FullSimplify}[\text{vp1} /. \text{Solve}[\text{EqVp} == 0, \text{vp1}][[1]]];$$

Note that the **Vb**’s, with capital “V”, store the solutions of the equations whereas the **vb**’s are the variables. This is why the **Vb**’s do not need to be defined at the beginning (see *In[1]*) of the notebook; they are implicitly defined here.

Now that we have the values of the perturbed velocities, we can derive the perturbed densities from Eqs. (3),

$$In[8]:= \text{Nb1} = \text{FullSimplify}[\omega \text{pb}^2 \frac{\mathbf{m}}{4\pi \mathbf{q}^2} \frac{\text{Dot}[\mathbf{k}, \text{Vb1}]}{\omega - \text{Dot}[\mathbf{k}, \mathbf{V0b}]}];$$

$$In[9]:= \text{Np1} = \text{FullSimplify}[\omega \text{pp}^2 \frac{\mathbf{m}}{4\pi \mathbf{q}^2} \frac{\text{Dot}[\mathbf{k}, \text{Vp1}]}{\omega - \text{Dot}[\mathbf{k}, \mathbf{V0p}]}];$$



Here again, we prepare the introduction of the reduced variables (7) by expressing the equilibrium beam and plasma electronic densities in terms of the beam and plasma electronic frequencies.

We can now have *Mathematica* calculate the current according to Eq. (5),

*In[10]:=*

**J=FullSimplify[q $\left(\omega_{\text{pp}}^2 \frac{m}{4\pi q^2} \mathbf{V}_{\text{p1}} + \omega_{\text{pb}}^2 \frac{m}{4\pi q^2} \mathbf{V}_{\text{b1}} + \text{Np1} \mathbf{V}_{0\text{p}} + \text{Nb1} \mathbf{V}_{0\text{b}}\right)$ ];**

We now have the symbolic expression of the current  $\mathbf{J}$ . In order to find the tensor  $\mathcal{T}$  yielding the dispersion equation, we need to explain first the current tensor. This is performed through,

*In[11]:=M=*

$$\begin{pmatrix} \text{Coefficient}[\mathbf{J}[[1]], \mathbf{E1x}] & \text{Coefficient}[\mathbf{J}[[1]], \mathbf{E1y}] & \text{Coefficient}[\mathbf{J}[[1]], \mathbf{E1z}] \\ \text{Coefficient}[\mathbf{J}[[2]], \mathbf{E1x}] & \text{Coefficient}[\mathbf{J}[[2]], \mathbf{E1y}] & \text{Coefficient}[\mathbf{J}[[2]], \mathbf{E1z}] \\ \text{Coefficient}[\mathbf{J}[[3]], \mathbf{E1x}] & \text{Coefficient}[\mathbf{J}[[3]], \mathbf{E1y}] & \text{Coefficient}[\mathbf{J}[[3]], \mathbf{E1z}] \end{pmatrix} ;$$

which just extract the tensor elements from the expression of  $\mathbf{J}$ . We now turn to Eq. (6) where we explain the tensor elements of the quantity  $c^2 \mathbf{k} \times (\mathbf{k} \times \mathbf{E}_1) + \omega^2 \mathbf{E}_1$ ,

*In[12]:=M0=c^2 Cross[k,Cross[k,E1]]+omega^2 E1 ;*

*In*[13]:=M1=

$$\begin{pmatrix} \text{Coefficient}[M0[[1]], E1x] & \text{Coefficient}[M0[[1]], E1y] & \text{Coefficient}[M0[[1]], E1z] \\ \text{Coefficient}[M0[[2]], E1x] & \text{Coefficient}[M0[[2]], E1y] & \text{Coefficient}[M0[[2]], E1z] \\ \text{Coefficient}[M0[[3]], E1x] & \text{Coefficient}[M0[[3]], E1y] & \text{Coefficient}[M0[[3]], E1z] \end{pmatrix};$$

We can finally express the tensor  $\mathcal{T}$  defined by  $\mathcal{T}(\mathbf{E})=0$  as

*In*[14]:=T=M1+4 i  $\pi$   $\omega$  M;

At this stage of the notebook, we could take the determinant of the tensor to obtain the dispersion equation. Let us first introduce the dimensionless variables (7) through,

*In*[15]:=T=T /. {Vp  $\rightarrow$  - $\alpha$  Vb, kz  $\rightarrow$   $\omega_{pp}$  Zz/Vb, kx  $\rightarrow$   $\omega_{pp}$  Zx/Vb,  $\omega_{pb}^2 \rightarrow \alpha \omega_{pp}^2$ ,  $\omega \rightarrow x \omega_{pp}$ ,  $\omega_b \rightarrow \Omega b \omega_{pp}$ };

and,

*In*[16]:=T=T /. {Vb  $\rightarrow \beta$  c}

*Mathematica* leaves here some  $\omega_{pp}$ 's which should simplify between each others. It is enough to perform

*In*[17]:=T=T /. { $\omega_{pp} \rightarrow 1$ };

and a simple

*In*[18]:=MatrixForm[FullSimplify[T]]

displays the result. The dispersion equation of the system is eventually ob-

tained through

$In[19]:= \mathbf{DisperEq} = \mathbf{Det}[\mathbf{T}]$

The notebook evaluation takes 1 minute on a Laptop running a 1.5 GHz Pentium Centrino under Windows XP Pro. This delay can be shortened down to 10 seconds by suppressing all the **FullSimplify** routines while leaving a **Simplify** $[\mathbf{T}]$  in entry 18, but the final result is much less concise and readable.

## 4 Comments and Conclusion

In this paper, we have described a *Mathematica* notebook performing the symbolic evaluation of the dielectric tensor of a beam plasma system. Starting from the linearized fluid equations, the notebook expresses the dielectric tensor, and eventually the dispersion equation, in terms of some usual dimensionless parameters. This notebook has been so far applied to the treatment of the temperature dependant non magnetized and magnetized problems (see Refs [19,20]). Indeed, the procedure is very easy to adapt to different settings.

When including beam or plasma temperatures, one adds a pressure term  $-\nabla P_j/n_j$  on the right hand side of the force equations (2). Setting then  $\nabla P_i = 3k_B T_i \nabla n_i$  [12,22] if dealing only with electron motion, one only needs to add to the notebook entries 4 and 5 the terms ( $\mathbf{i}^2 = -1$ )

$$-3\mathbf{i} \mathbf{Tj} \mathbf{k} \frac{\mathbf{Dot}[\mathbf{k}, \mathbf{vj1}]}{\omega - \mathbf{Dot}[\mathbf{k}, \mathbf{V0j}]},$$

where  $\mathbf{j} = \mathbf{p}$  for the plasma, and  $\mathbf{b}$  for the beam. When considering anisotropic temperatures [19], one just needs to define a temperature tensor  $\mathbf{Tj}$  for each species  $\mathbf{j}$ , and replace the scalar product  $\mathbf{Tj} \mathbf{k}$  by the tensorial one  $\mathbf{Dot}[\mathbf{Tj}, \mathbf{k}]$

in both entries. Of course, a correct treatment of electromagnetic instabilities generally requires a kinetic formalism instead of a fluid one. However, kinetic calculations cannot be systematically entrusted to *Mathematica*, as is the case here. The reason why is that the relativistic factors  $\gamma$  encountered in the kinetic quadratures are coupling the integrations along the three components of the momentum. According to the distribution functions considered, the quadratures may be calculable through some ad hoc change of variables, if they can be calculated at all. At any rate, the process cannot be systematized enough for *Mathematica* to handle it.

As far as the magnetic field is concerned, its direction can be changed from entry *1* without any modification of the next entries. When dealing with the motion of ions, or even with one of these pair plasmas involved in the pulsar problems [23], one just need to modify the conservation and force equations according to the properties of the species investigated. It is even possible to add more equations to account for more species because the resolution involves only the force and the conservation equations of one single specie at a time before the perturbed quantities merge together in entry *10* to compute the current **J**.

The notebook can thus be easily adapted to different settings and allows for a quick symbolic calculation of the dielectric tensor and the dispersion equation, even for an elaborated fluid model.

## 5 Acknowledgements

This work has been achieved under projects FTN 2003-00721 of the Spanish Ministerio de Educación y Ciencia and PAI-05-045 of the Consejería de Educación y Ciencia de la Junta de Comunidades de Castilla-La Mancha.

## References

- [1] M. Tabak, J. Hammer, M. E. Glinsky, W. L. Kruer, S. C. Wilks, J. Woodworth, E. M. Campbell, M. D. Perry, and R. J. Mason. *Phys. Plasmas*, 1:1626, 1994.
- [2] M. Tabak, D. S. Clark, S. P. Hatchett, M. H. Key, B. F. Lasinski, R. A. Snavely, S. C. Wilks, R. P. J. Town, R. Stephens, E. M. Campbell, R. Kodama, K. Mima, K. A. Tanaka, S. Atzeni, and R. Freeman. *Phys. Plasmas*, 12(5):057305, 2005.
- [3] R. Fender and T. Belloni. *Ann. Rev. Astron. Astrophys.*, 42:317, 2004.
- [4] J.A. Zensus. *Ann. Rev. Astron. Astrophys.*, 35:607, 1997.
- [5] T. Piran. *Rev. Mod. Phys.*, 76:1143, 2004.
- [6] Y.A. Gallant and J. Arons. *Astrophys. J.*, 435:230, 1997.
- [7] Ya. B. Faĭnberg, V.D. Shapiro, and V.I. Shevchenko. *Soviet Phys. JETP*, 30:528, 1970.
- [8] B. B. Godfrey, W. R. Shanahan, and L. E. Thode. *Phys. Fluids*, 18:346, 1975.
- [9] F. Califano, F. Pegoraro, and S. V. Bulanov. *Phys. Rev. E*, 56:963, 1997.
- [10] F. Califano, F. Pegoraro, S. V. Bulanov, and A. Mangeney. *Phys. Rev. E*, 57:7048, 1998.

- [11] F. Califano, R. Prandi, F. Pegoraro, and S. V. Bulanov. *Phys. Rev. E*, 58:7837, 1998.
- [12] M. Honda. *Phys. Rev. E*, 69:016401, 2004.
- [13] A. Bret, M.-C. Firpo, and C. Deutsch. *Phys. Rev. E*, 70:046401, 2004.
- [14] A. Bret, M.-C. Firpo, and C. Deutsch. *Phys. Rev. E*, 72:016403, 2005.
- [15] A. Bret, M.-C. Firpo, and C. Deutsch. *Phys. Rev. Lett.*, 94:1150024, 2005.
- [16] J.R. Cary, L.E. Thode, D.S. Lemons, M.E. Jones, and M.A. Mostrom. *Phys. Fluids*, 24:1818, 1981.
- [17] R.C. Tautz and R. Schlickeiser. *Phys. Plasmas*, 12:122901, 2005.
- [18] R.C. Tautz, I. Lerche, and R. Schlickeiser. *Phys. Plasmas*, 13:052112, 2006.
- [19] A. Bret and C. Deutsch. *Phys. Plasmas*, 13:042106, 2006.
- [20] A. Bret, M. Dieckmann, and C. Deutsch. *Phys. Plasmas*, 13:082109, 2006.
- [21] S. Ichimaru. *Basic Principles of Plasma Physics*. W. A. Benjamin, Inc., Reading, Massachusetts, 1973.
- [22] W.L. Kruer. *The Physics of Laser Plasma Interaction*. Westview Press, Boulder, Colorado, 2003.
- [23] M. Gedalin, E. Gruman, and D.B. Melrose. *Phys. Rev. Lett.*, 88:121101, 2002.